

## LA-UR-21-22827

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Title: iFP: A Multiscale Eulerian Vlasov-Fokker-Planck Code for Modeling Inertial Confinement Fusion Capsules

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Intended for: Internal LANL seminar at the Multiscale Multiphysics seminar

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Issued: 2021-03-23

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# iFP: A Multiscale Eulerian Vlasov-Fokker-Planck Code for Modeling Inertial Confinement Fusion Capsules

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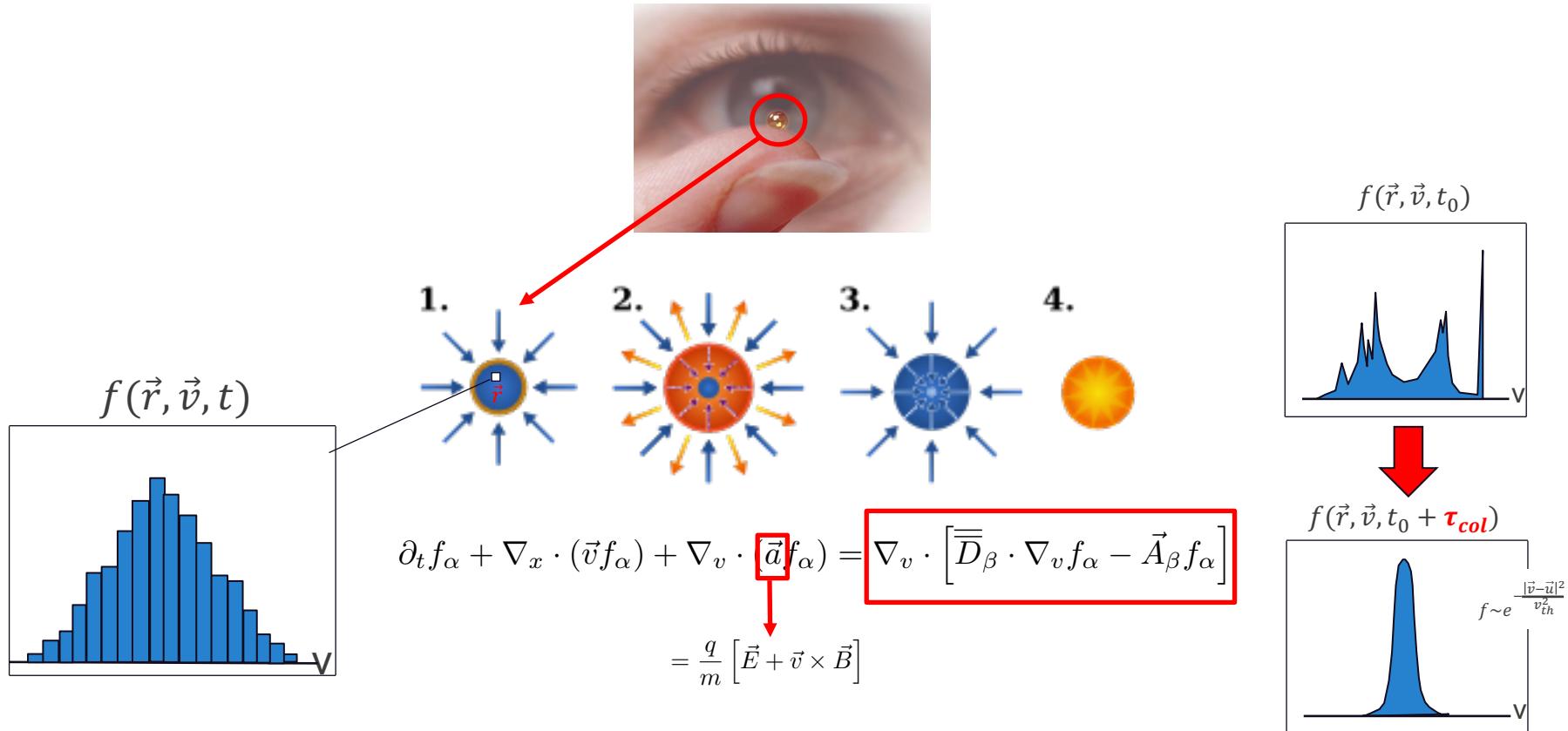
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# Vlasov-Fokker-Planck equation and applications to ICF



# Macroscopic (hydro) approaches can probe a fraction of the entire HED parameter space and kinetic models are required to access broader range

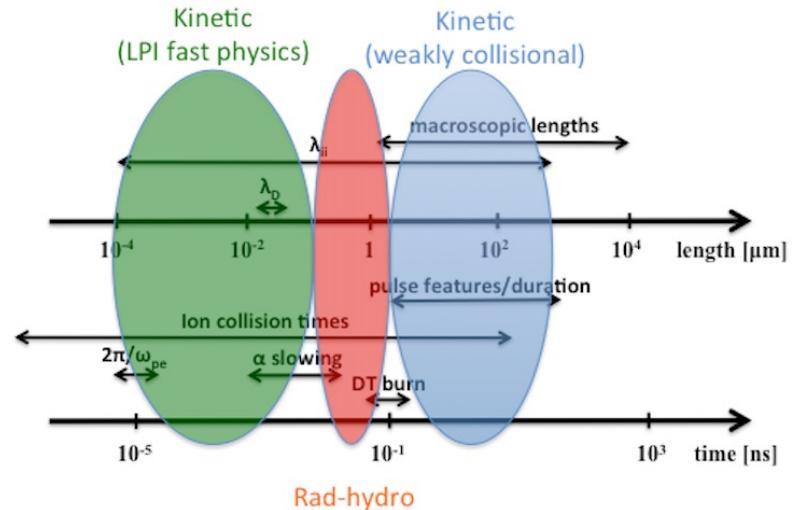
Hydrodynamic (Euler) equations:

$$\frac{\partial \rho}{\partial t} + \nabla_x \cdot \rho \vec{u} = 0$$

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla_x \cdot [\rho \vec{u} \otimes \vec{u} + \bar{\bar{I}}P] = 0$$

$$\frac{\partial \rho e_t}{\partial t} + \nabla_x \cdot [\vec{u} (\rho e_t + P)] = 0$$

a particular asymptotic limit of VFP for  $\lambda_{col} \ll L$  and  $\tau_{col} \ll \tau$

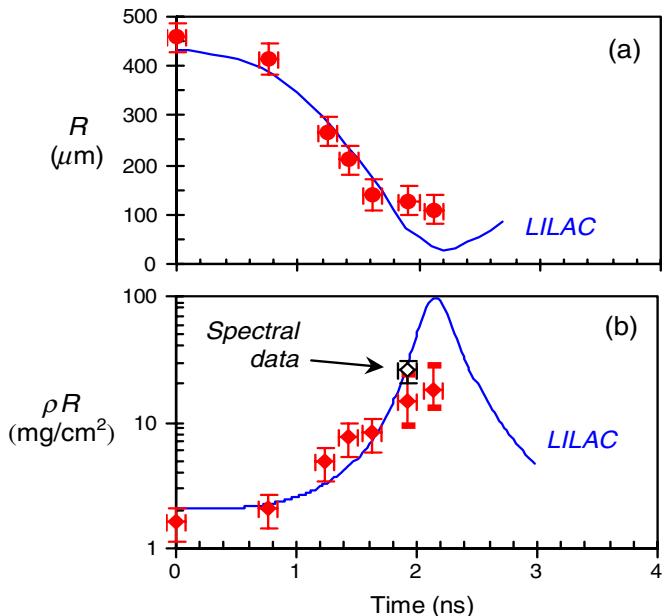


# ICF implosions are riddled with plasma kinetic effects

- **LPI effects (*fundamentally kinetic*)**
  - Backscattered and “sprayed” laser energy
  - Cross-beam energy transfer (CBET)
- **Multi-ion effects due to differential motion and heating of various ion species (*exist in fluid limit, but are strongly enhanced by kinetic effects*)**
  - Capsule’s fuel composition modifications (fuel segregation)
  - Mix at interfaces
  - Ion interpenetration and nonlocal electron transport in Hohlraums
- **Recent OMEGA campaigns and other dedicated experimental campaigns have highlighted serious deficiencies in our ability to**
  - Predict capsule **compression and yield** (both are over-predicted)
  - Predict **time-dependent core mix** (**especially when hydro instabilities** are not expected to play a role)

# Compression: Often over-predicted by hydro simulations

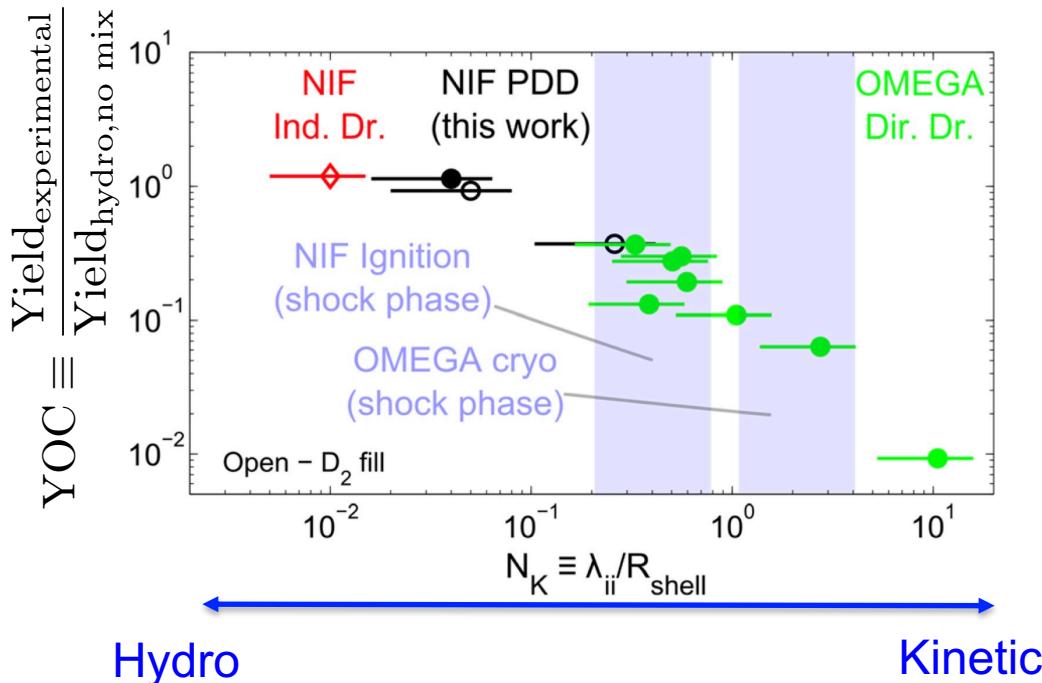
20- $\mu\text{m}$  thick CH shell, 15 atm.  $\text{H}_2$  fill



Li *et al.*, PRL 100, 225001 (2008)

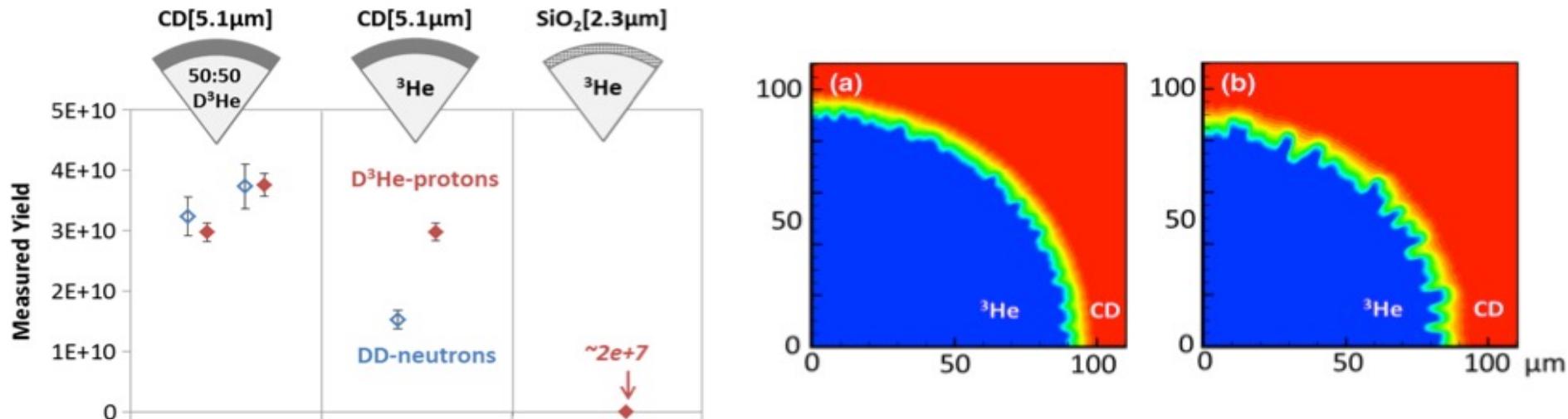
# Yield: Rad-hydro fails to capture yield decrease with decreasing fuel fill pressure

Rosenberg *et al*, PRL 112 (2014)



# Mix: Exploding pusher experiments demonstrate ~10% fuel-shell mix *prior* to deceleration (no hydro growth)

## Deuterated capsules



Rinderknecht et al., PRL 112, 135001 (2014)

# Numerical Challenges

# The equation is high-dimensional, integro-differential and heavily constrained (conservation and positivity)

$$\partial_t f_\alpha + \vec{v} \cdot \nabla_x f_\alpha + \vec{a}_\alpha \cdot \nabla_v f_\alpha = \nabla_v \cdot \left[ \overline{\overline{D}}_\beta \cdot \nabla_v f_\alpha - \frac{m_\alpha}{m_\beta} \vec{A}_\beta f_\alpha \right]$$

$$\overline{\overline{D}}_\beta = \nabla_v \nabla_v G_\beta \quad \vec{A}_\beta = \nabla_v H_\beta$$

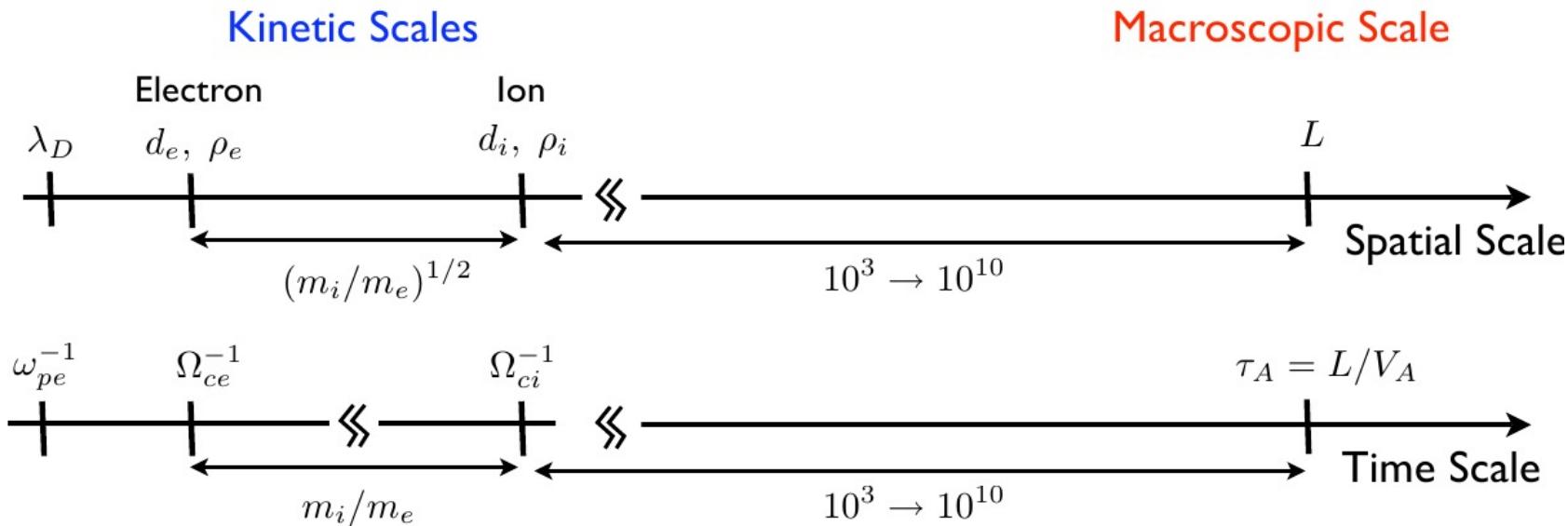
$$\nabla_v^2 G_\beta = H_\beta \quad \nabla_v^2 H_\beta = -8\pi f_\beta \quad \text{integral nature}$$

$$\left\langle \begin{bmatrix} \vec{v} \\ v^2 \end{bmatrix}, C_{\alpha\beta} + C_{\beta\alpha} \right\rangle_v = \vec{0} \quad f_\alpha \geq 0 \ \forall \{\vec{r}, \vec{v}, t\} \quad \text{constraints}$$

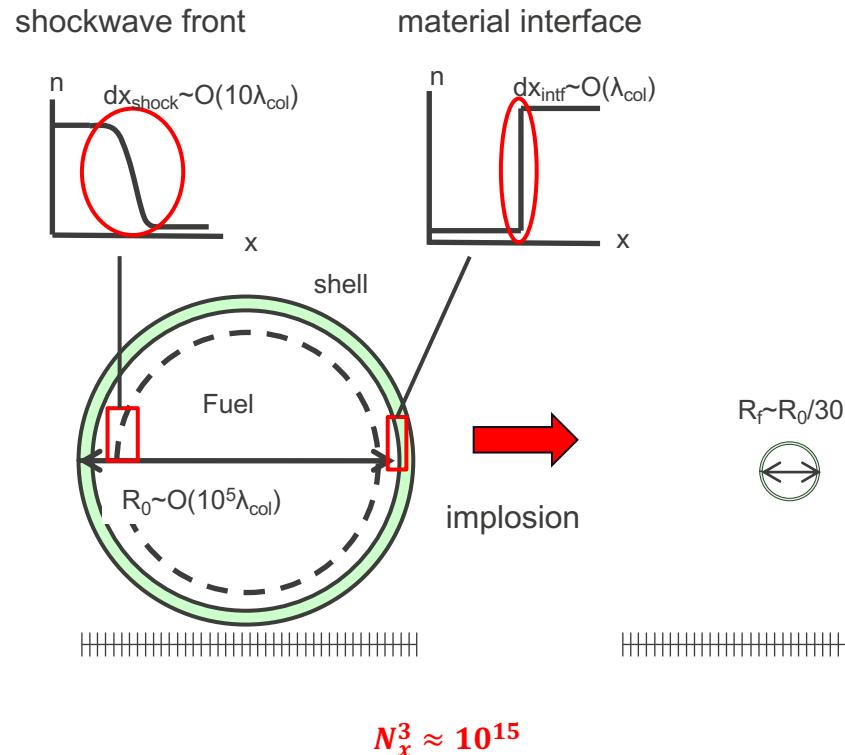
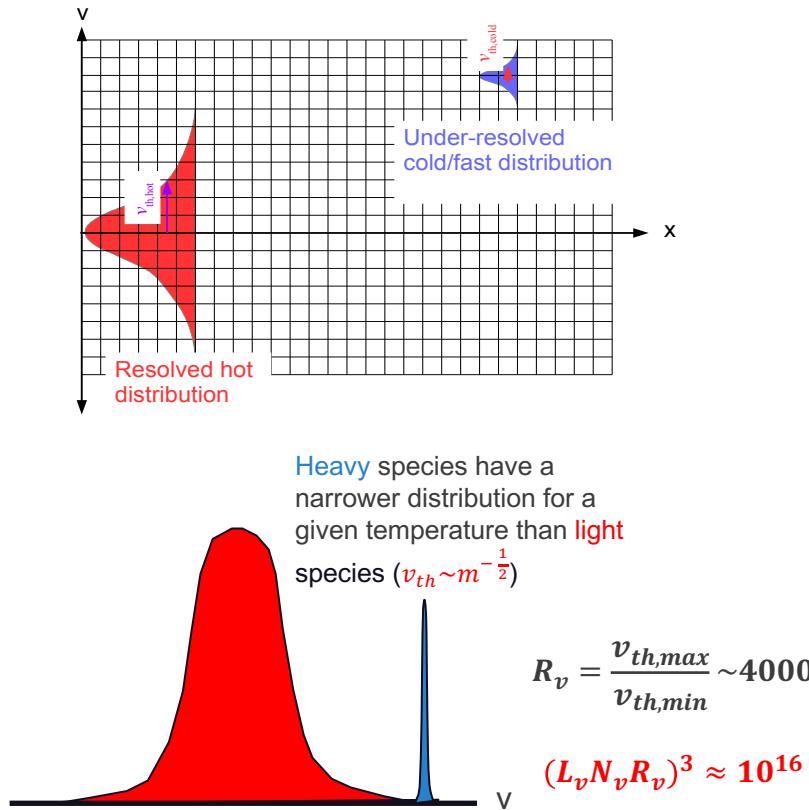
+ Maxwell's equations...

High dimensionality (3D+3V), exceedingly multiscale

However, system-scale kinetic simulation requires several enabling capabilities that can **efficiently** and **accurately** span scales



# Phase-Space Curse of Dimensionality : Kinetic description introduces additional challenges in dealing with disparate phase-space scales



# The collisional diffusion operator imposes extreme constraints on explicit time-integrators

$$\Delta t_{exp} \propto \tau_{col,pusher} \Delta v^2 \sim 10^{-12} \text{ [ns]} \text{ (dynamically irrelevant)}$$

$$\tau_{col,push} \sim 10^{-8} \text{ [ns]} \text{ (dynamically irrelevant)}$$

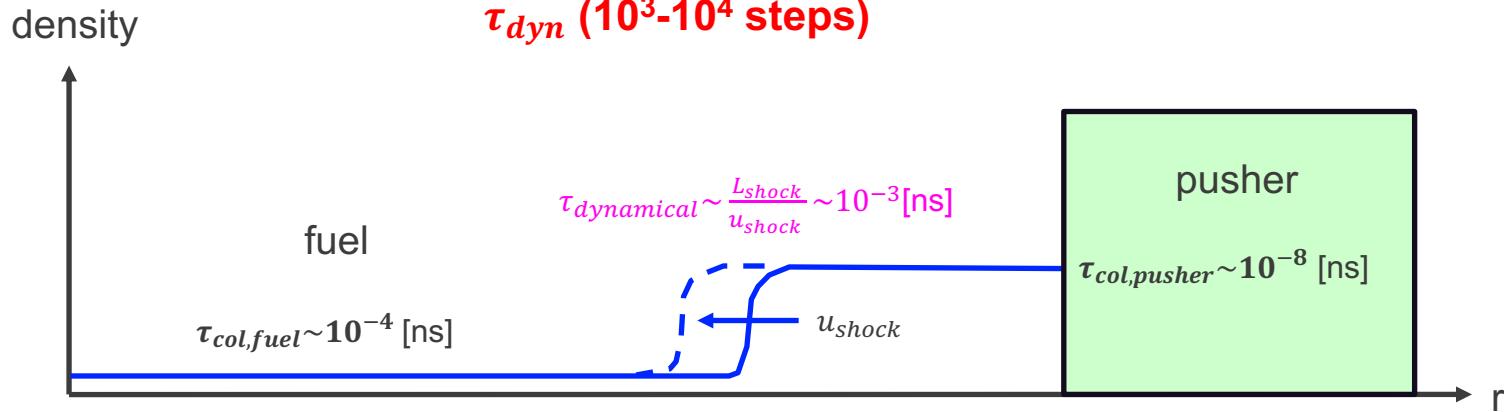
$$\tau_{col,fuel} \sim 10^{-4} \text{ [ns]}$$

$$\tau_{dynamical} \sim 10^{-3} \text{ [ns]}$$

$$t_{sim} = 1 \sim 10 \text{ [ns]}$$

**$N_t \geq 10^{12} - 10^{13}$  with explicit time integration  
even though relevant physics only happens on**

**$\tau_{dyn}$  ( $10^3$ - $10^4$  steps)**

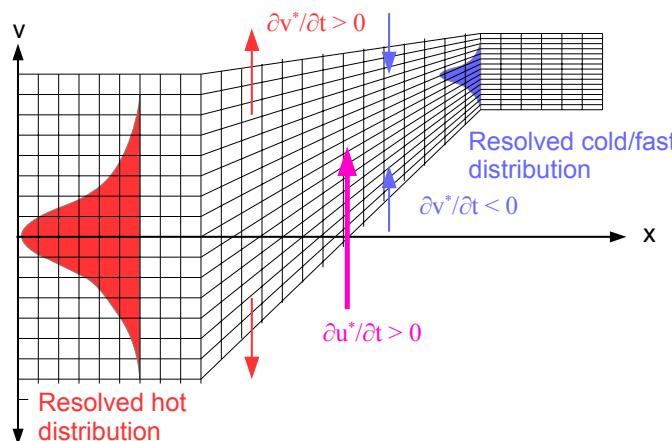


# **Physics-based moving phase-space grid strategy with discrete mass, momentum, and energy conservation**

# We have explored moving phase-space grid strategy in 1D2V to address the length and velocity scale challenges

Velocity space transformation:  $\vec{v} = v^* \vec{\tilde{v}} + \vec{u}^*$

$v^* \sim v_{th}$  (variance of the distribution)  
 $\vec{u}^* \sim \vec{u}$  (the mean of the distribution)

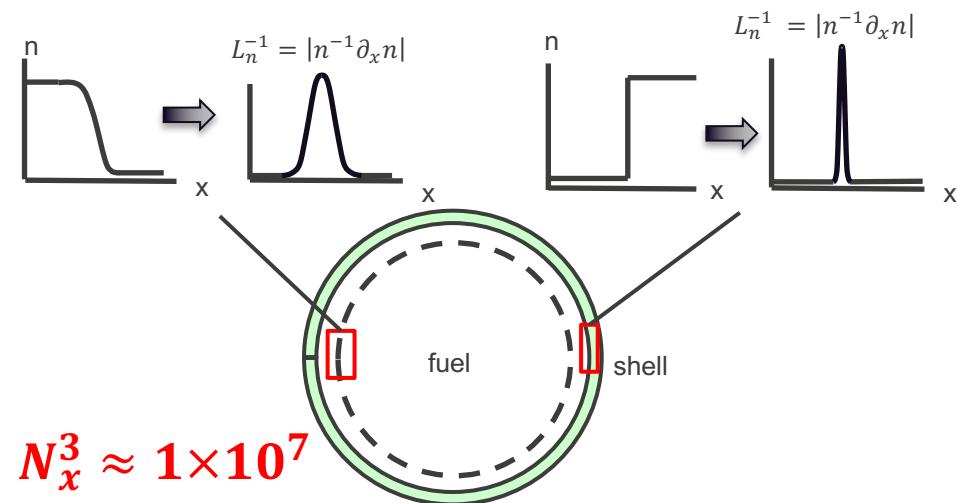


$$(L_v N_v)^3 \approx 1 \times 10^5$$

We solve for x-grid position based on error equidistribution principle with a nonlinear optimization step to stabilize grid evolution:

$$\partial_t x^* = \tau_g^{-1} \partial_\xi [L_n^{-1} \partial_\xi x^*]$$

$\min \mathbb{F}(x^*) \rightarrow x$  where  $\mathbb{F}$  is a cost function which leads to a corrected grid,  $x$ , which leads to an optimal balance between grid quality and grid evolution



$$N_x^3 \approx 1 \times 10^7$$

# All this amounts to transforming the VFP equation into the new curvilinear coordinate system with additional inertial terms

$$\partial_t f_\alpha + \nabla_x \cdot (\vec{v} f_\alpha) + \nabla_v \cdot (\vec{a} f_\alpha) = \sum_{\beta}^{N_s} \nabla_v \cdot [\vec{J}_{\alpha\beta}]$$

↓

$$\partial_t (\mathbb{J} f_\alpha) + \nabla_\xi \cdot [\mathbb{J} (\vec{v}^\xi - \vec{r}^\xi) f_\alpha] + \nabla_{\tilde{v}} \cdot [\mathbb{J} (\vec{\tilde{a}} - \vec{\tilde{a}}^+) f_\alpha] = \sum_{\beta}^{N_s} \nabla_{\tilde{v}} \cdot [\mathbb{J} \vec{\tilde{J}}_{\alpha\beta}]$$

inertial terms/Christoffel symbols

- $\mathbb{J}$ : Jacobian of transformation
- $\vec{r}^\xi$ : contravariant component of configuration grid velocity
- $\vec{v}^\xi$ : contravariant component of particle velocity
- $\vec{\tilde{a}}$ : Lorentz force in the new velocity coordinate system
- $\vec{\tilde{a}}^+$ : Fictitious acceleration terms due to expansion/contraction of velocity grid

# But naïve discretization of transformed equation breaks discrete conservation principle (conservation holds in the continuum)

- Case study (total energy conservation):

- Single species
- Spatially homogeneous (0D)
- No background EM field
- Maxwellian distribution (null-space of FP)
- But with  $v^*$  and  $\vec{u}^*$  changing in time

$$\partial_t (Jf) - \nabla_{\tilde{v}} \cdot \left[ J \tilde{a}^\dagger f \right] = 0$$

↓  
 $\partial_t \vec{v}$   
↓  
 $\partial_t (v^* \vec{v} + \vec{u}^*)$

Defining:

$$\langle A(\vec{v}), B(\vec{v}) \rangle_v = \int A(\vec{v})B(\vec{v}) d^3 v$$

The total energy of system is defined as:

$$JU = \left\langle \frac{\vec{v} \cdot \vec{v}}{2}, Jf \right\rangle_v$$

Energy conservation is defined as:  
 $\partial_t (JU) = 0$

We require that the following chain rule is satisfied  
**in the discrete:**

$$\begin{aligned} & \langle \vec{v} \cdot \vec{v}, \partial_t (Jf) - \nabla_{\tilde{v}} \cdot [J(\partial_t \vec{v})f] \rangle_v = \\ & \quad \langle 1, \partial_t (\vec{v} \cdot \vec{v}, Jf) \rangle_v \end{aligned}$$

# For the time being, we rely on the method of discrete nonlinear constraints to ensure that discrete conservation symmetries are enforced

In general, an arbitrary discretization leads to:

$$\begin{aligned} & \langle \vec{v} \cdot \vec{v}, \delta_t(Jf) - \vec{\delta}_{\vec{v}} \cdot [J(\delta_t \vec{v})f] \rangle_{\delta v} = \\ & \quad \langle 1, \delta_t(\vec{v} \cdot \vec{v}, Jf) \rangle_{\delta v} + O(\Delta v^\beta, \Delta t^\eta) \end{aligned}$$

Add an additional degree of freedom:

$$\partial_t(Jf) - \nabla_{\vec{v}} \cdot [\gamma J(\partial_t \vec{v})f] = 0$$

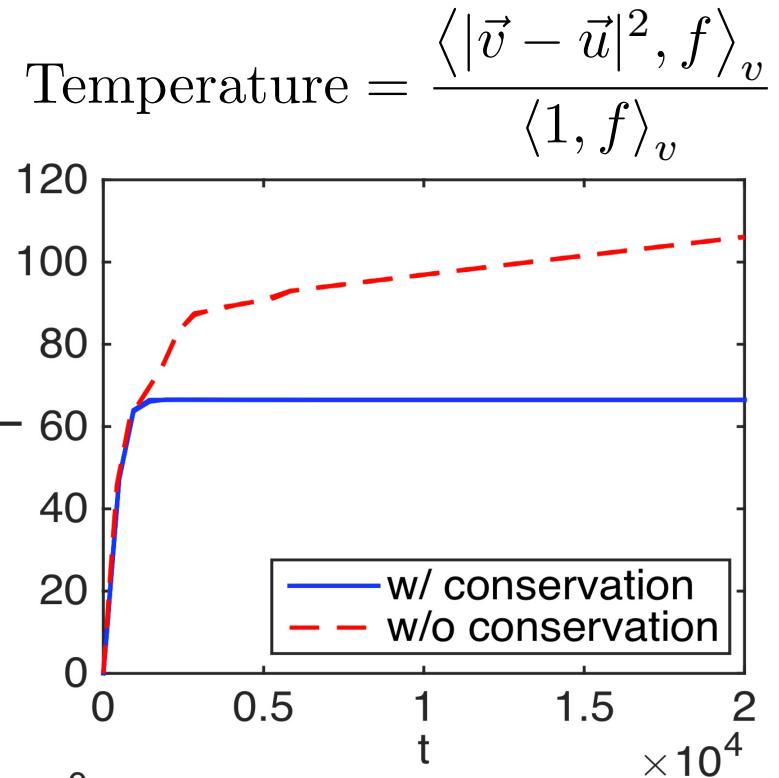
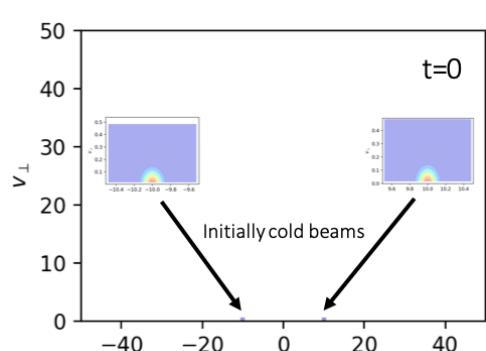
$$\text{where } \gamma = 1 + O(\Delta v^\beta, \Delta t^\eta)$$

Such that, we discretely satisfy the continuum symmetry/constraint:

$$\langle \vec{v} \cdot \vec{v}, \delta_t(Jf) - \vec{\delta}_{\vec{v}} \cdot [\gamma J(\delta_t \vec{v})f] \rangle_{\delta v} = \langle 1, \delta_t(\vec{v} \cdot \vec{v}, f) \rangle_{\delta v}$$

The use of skew brackets are currently in the planning to ensure automatic (structure preserving) conservation properties of the inertial terms

# Lack of conservation can lead to catastrophic failure of simulations



# **HOLO accelerated fully nonlinearly implicit solver**

# Nonlinearly implicit solvers can stably and accurately step over stiff time-scales

$\Delta t_{exp} \propto \tau_{col,pusher} \Delta v^2 \sim 10^{-12}$  [ns] (but dynamically irrelevant)

$\tau_{dynamical} \sim 10^{-3}$  [ns]

$t_{sim} = 1 \sim 10$  [ns]

$N_t \geq 10^{14} - 10^{15}$  with explicit time integration

**$N_t \sim 10^3 - 10^4$  with fully implicit solvers**

# Need nonlinearly implicit method to step over stiff $\tau_{\text{col}}$ BUT with a catch...

- Both Newton and (accelerated) Picard requires a Jacobian/map:

$$f^{k+1} = f^k + \delta f^k \quad P^k \delta f^k = -R^k$$

- If  $P = J$ , Newton and if  $P \approx J$ , Picard:

$$R = \partial_t f - \nabla_v \cdot [\bar{D} \cdot \nabla_v f - \vec{A} f]$$

$$J \delta f = \frac{\partial R}{\partial f} \delta f = \partial_t \delta f - \nabla_v \cdot \left[ \left( \bar{D} \cdot \nabla_v \delta f - \vec{A} \delta f \right) + \left( \frac{\partial \bar{D}}{\partial f} \cdot \nabla_v f - \frac{\partial \vec{A}}{\partial f} f \right) \delta f \right]$$

Local (sparse)

Non-local (dense) very  
difficult to deal with

- Typically, one only retains the local piece [bad if  $\Delta t > \tau_{\text{col}}$  (nonlinear iteration grows out of control)]

# we exploit the relationship between kinetic and moment equations

Kinetic system (3D3V)

$$\mathbb{B}_\alpha = \partial_t f_\alpha + \vec{v} \cdot \nabla_x f_\alpha - \sum_{\beta}^{N_s} C(f_\beta, f_\alpha) = 0$$

consistent representation



Moment system (3D)

$$\langle m, \mathbb{B}_\alpha \rangle_{\vec{v}} = \partial_t \rho_\alpha + \nabla_x \cdot (\rho \vec{u}_\alpha) = 0$$

$$\langle m \vec{v}, \mathbb{B}_\alpha \rangle_{\vec{v}} = \partial_t \rho \vec{u}_\alpha + \nabla_x \cdot (\bar{\overline{S}}_{2,\alpha}) - \sum_{\beta}^{N_s} F_{\alpha\beta} = 0$$

$$\left\langle m \frac{v^2}{2}, \mathbb{B} \right\rangle_{\vec{v}} = \partial_t \rho U_\alpha + \nabla_x \cdot (\vec{S}_{3,\alpha}) - \sum_{\beta}^{N_s} W_{\alpha\beta} = 0$$

⋮

consistent physics

for collisionally dominated system, stiff physics are captured by first three moments (Navier Stokes)

closure

$$\langle m v^l, \mathbb{B}_\alpha \rangle_{\vec{v}} = \partial_t \mathcal{M}_\alpha^l + \nabla_x \cdot \vec{S}_l - \sum_{\beta}^{N_s} \mathbb{W}_{l,\alpha\beta} = 0$$

**Key: Decompose operators in terms of stiff hydrodynamic (LTE) component and perturbation.**  
**Let LO system deal with stiffness while HO system only deals with perturbations, and provide closures to the LO system**

## HO Equation

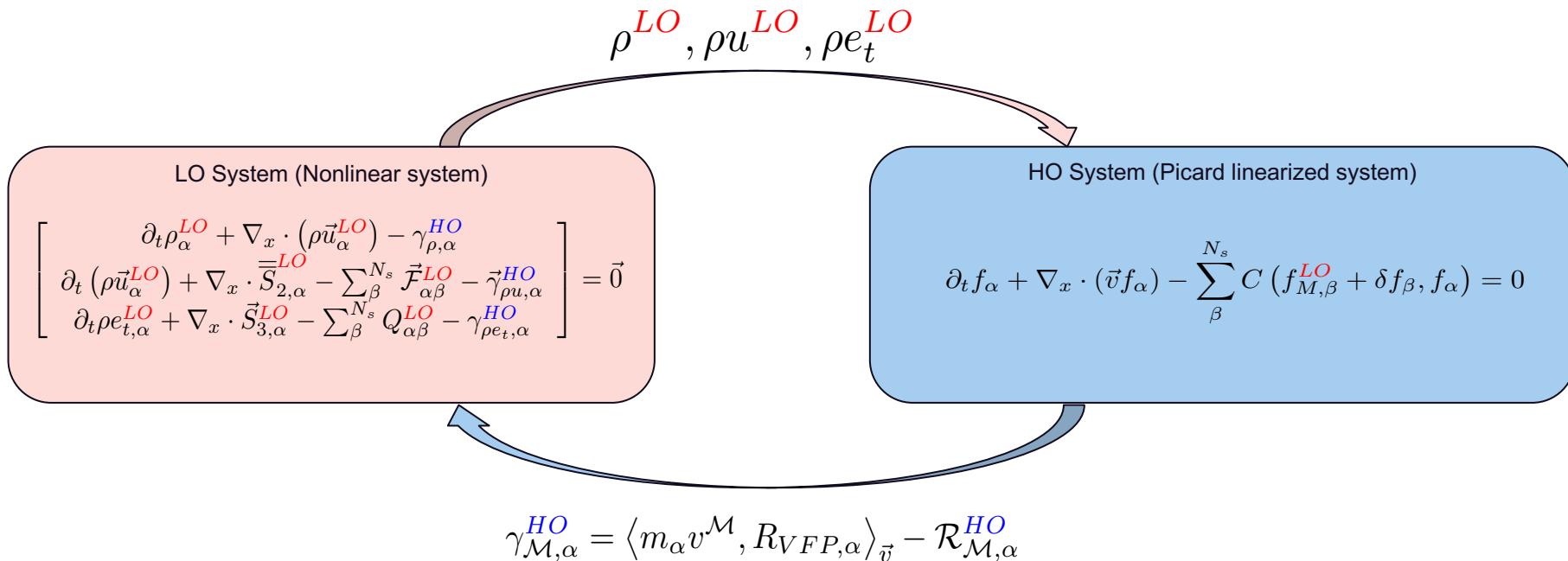
$$VFP_\alpha = \partial_t f_\alpha + \vec{v} \cdot \nabla_x f_\alpha - \sum_\beta \left\{ \underbrace{\nabla_v \cdot \left[ \overline{\overline{D}}_\beta^{M,LO} \cdot \left( \nabla_v \ln f_\alpha - \nabla \ln f_M \left( \vec{v}; (\mathbf{n}_\alpha, \vec{u}_\beta, T_\beta)^{LO} \right) f_\alpha \right) \right]}_{O(\frac{1}{\epsilon}) \text{ the stiff piece (difficult)}} + \underbrace{\nabla_v \cdot \left[ \left( \overline{\overline{\delta D}}_\beta \cdot \nabla_v \ln f_\alpha - \overrightarrow{\delta A}_\beta \right) f_\alpha \right]}_{O(\epsilon) \text{ the non-stiff piece (easy)}} \right\} = 0$$

$$\epsilon = \frac{\tau_{col}}{\Delta t}$$

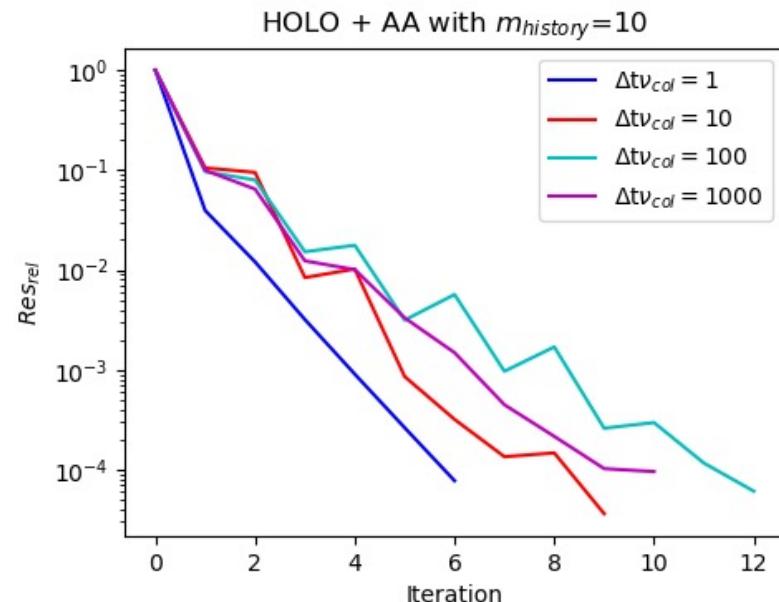
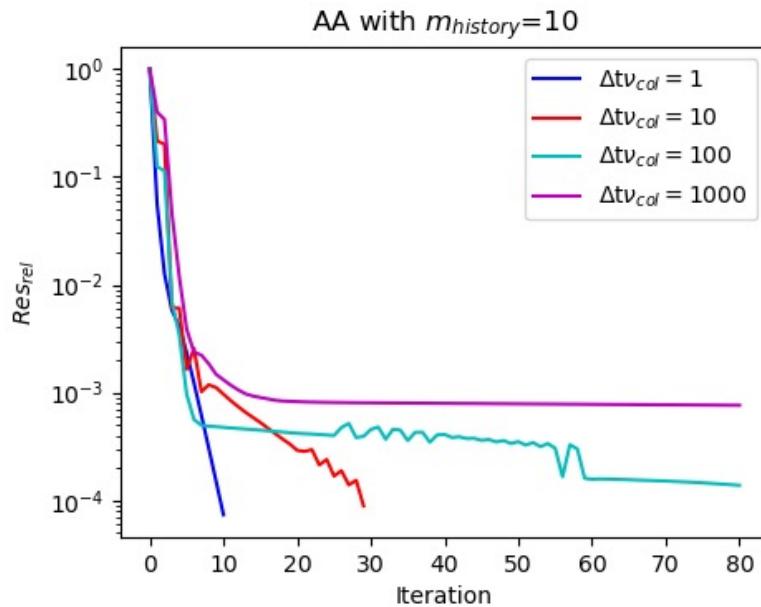
## LO Equation

$$\left\langle m \begin{bmatrix} 1 \\ \vec{v} \\ v^2/2 \end{bmatrix}, VFP_\alpha \right\rangle_{\vec{v}} \Rightarrow \vec{\mathcal{R}}_{\mathcal{M},\alpha}^{LO} = \begin{bmatrix} \partial_t \rho_\alpha^{LO} + \nabla_x \cdot \rho \vec{u}_\alpha^{LO} - \gamma_{\rho,\alpha}^{HO} \\ \partial_t \rho \vec{u}_\alpha^{LO} + \nabla_x \cdot \overline{\overline{S}}_{2,\alpha}^{LO} - \sum_\beta^{N_s} \vec{\mathcal{F}}_{\alpha\beta}^{LO} - \vec{\gamma}_{\rho u,\alpha}^{HO} \\ \partial_t \rho e_{t,\alpha}^{LO} + \nabla_x \cdot \vec{S}_{3,\alpha}^{LO} - \sum_\beta^{N_s} Q_{\alpha\beta}^{LO} - \gamma_{\rho e t,\alpha}^{HO} \end{bmatrix} = \vec{0}$$

# Iterate HO and LO system to convergence



# HOLO truly enables integrated ICF type problems by being able to efficiently step over dynamically irrelevant stiff collision time scales



AA (m=10): Anderson acceleration with  $m=10$   
nonlinear histories on a Quasi-Newton (sparse  
Jacobian representation) fixed point iteration scheme

# **Physics demonstration: ICF exploding pusher problem**

# iFP: A hybrid 1D2V VFP code for spherical implosion simulations of ICF capsules



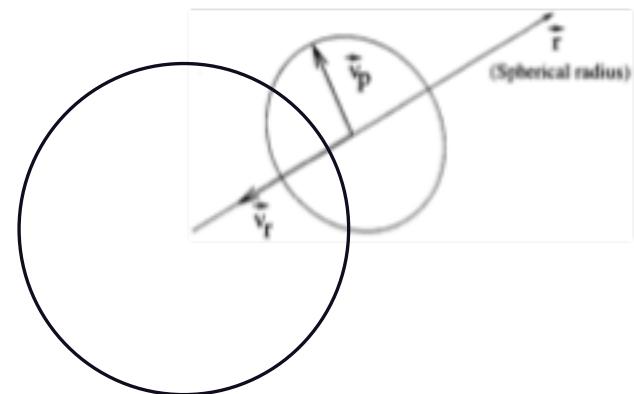
- Hybrid VFP ion (arbitrary species) and fluid electron with quasi-neutrality and ambipolarity

$$\partial_t f_\alpha + \nabla_x \cdot (\vec{v} f_\alpha) + \nabla_v \cdot (\vec{a} f_\alpha) = \nabla_v \cdot \left[ \overline{\overline{D}}_\beta \cdot \nabla_v f_\alpha - \vec{A}_\beta f_\alpha \right]$$

$$\frac{3}{2} \partial_t P_e + \frac{5}{2} \nabla_x \cdot (\vec{u}_e P_e) - \vec{u}_e \cdot \nabla_x P_e = \sum_{\alpha}^{N_s} W_{e\alpha}$$

$$n_e = \sum_{\alpha}^{N_s} q_{\alpha} n_{\alpha} / q_e \quad \vec{u}_e = \sum_{\alpha}^{N_s} n_{\alpha} \vec{u}_{\alpha} / q_e n_e$$

- 1D2V spherical radial and cylindrical velocity space
- Electrostatic (Ohm's law)



# Omega 95500 exploding pusher experiment

collaboration with Owen Mannion (U. Rochester), Brian Appelbe and Aidan Crilly (Imperial College)

- Experiments have seen anomalous nuclear yield reduction and lower reaction inferred ion temperatures (than radhydro)

Obs.	Rad hydro	Experiment (95500)
$T_{DD}$ (keV)	21.3	$13.1 \pm 1.25$
$T_{DT}$ (keV)	19.9	$14.2 \pm 0.40$
$Y_{DD}$	$1.77 \times 10^{11}$	$1.44 \times 10^{10}$
$Y_{DT}$	$3.67 \times 10^{13}$	$1.54 \times 10^{13}$

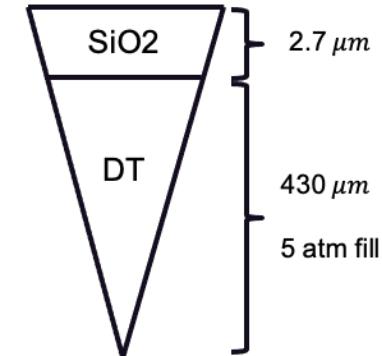
- Experimental conditions support strong kinetic effects

$$Kn = \frac{\lambda_{mfp}}{R_{capsule}} \geq 1 \text{ for an appreciable duration of the simulation}$$

- Good integrated problem for demonstrating combined strategy

$$\frac{\tau_{dyn}}{\Delta t_{exp}} > 10^7$$

$$(N_v N_x)_{static} \sim 10^{11}$$

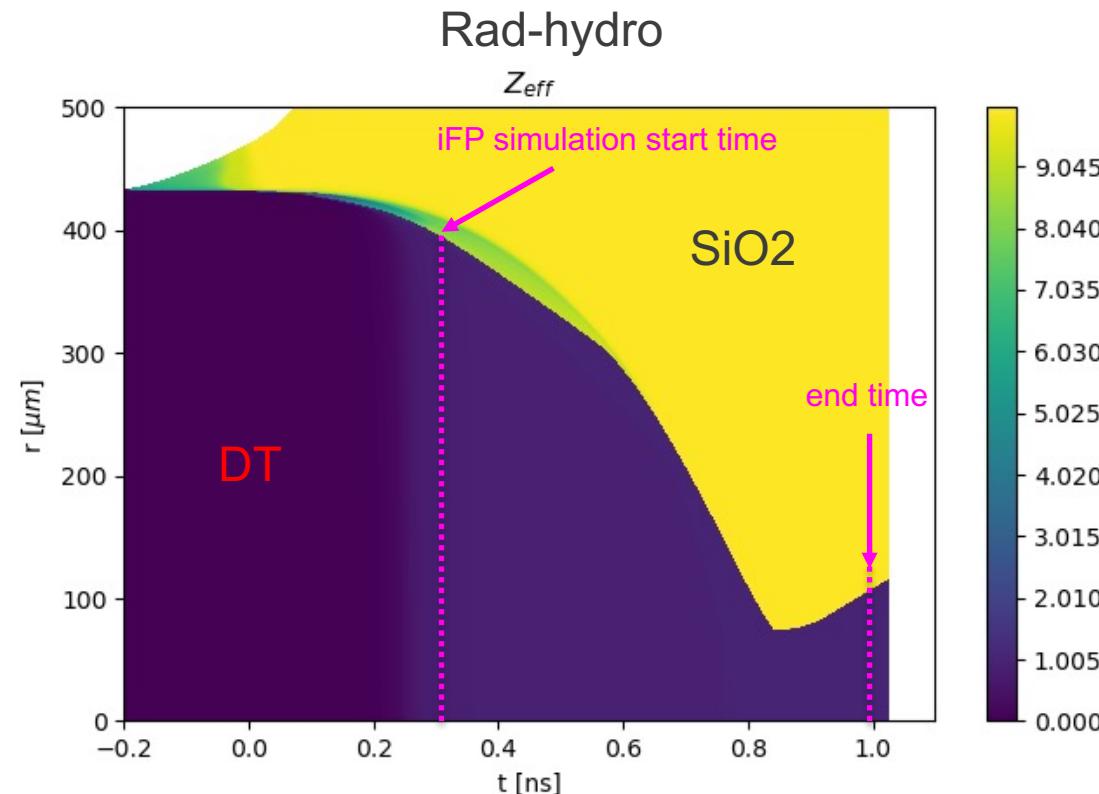


# iFP simulations is initialized using Radhydro solution at time when most of fuel fully ionized

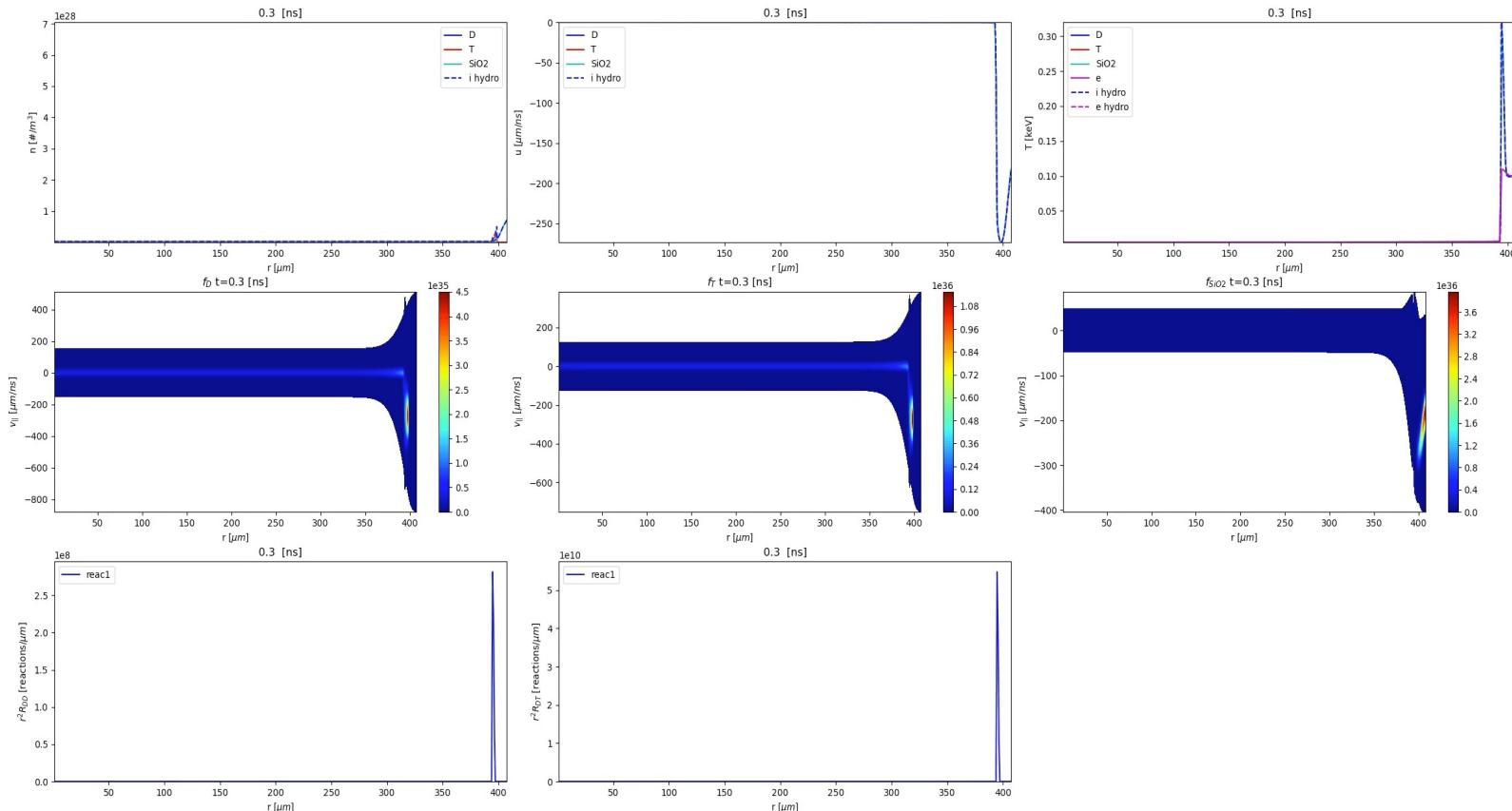
- iFP is driven by time-dependent Maxwellian with rad-hydro state variables:

$$f_{iFP,B} = \frac{n_H}{\pi^{3/2} v_{th,H}^3} e^{-\frac{(\vec{v}-\vec{u}_H)^2}{v_{th,H}^2}}$$

- $n_H, u_H, v_{th,H}$  obtained from radhydro simulation at specified Lagrangian zone inside the pusher

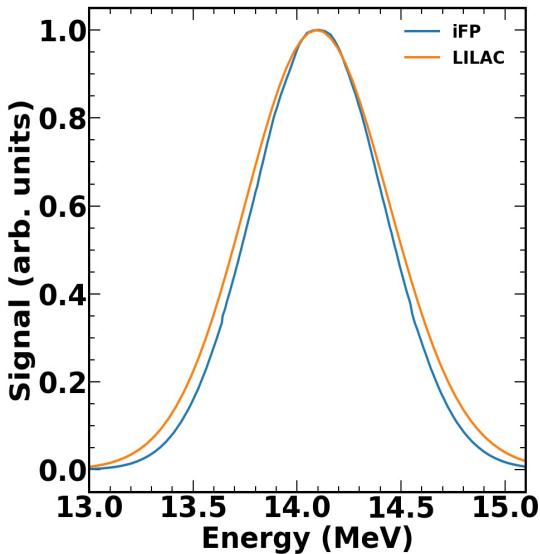


# Significant amount of SiO<sub>2</sub> ablator mix to center of fuel

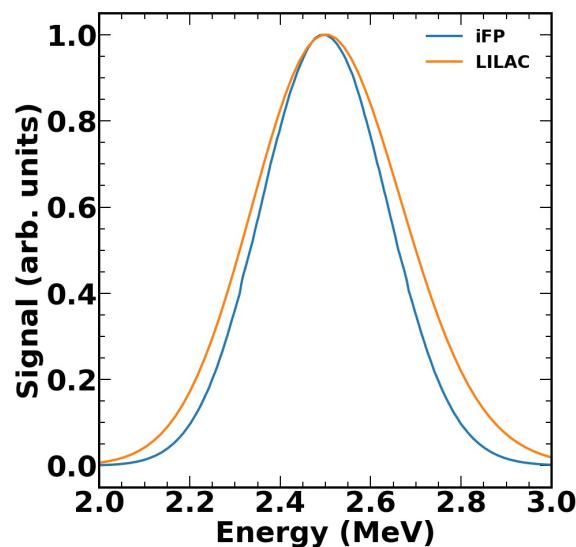


# kinetic effects alters implosion dynamics and modifies neutron spectrum

DT Neutron energy spectrum



DD Neutron energy spectrum

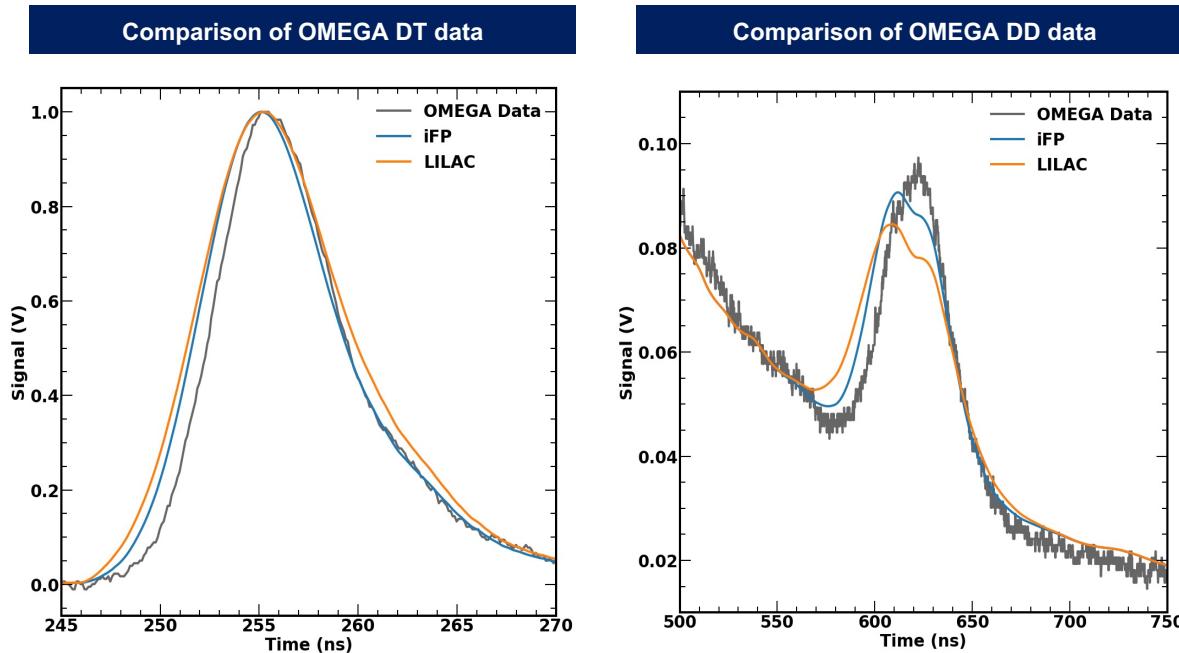


Apparent ion temperature of the plasma is inferred from the width of these spectra

# kinetic effects captures order of magnitude correction in yield and brings inferred temperature closer to experiment

Obs.	LILAC (Rad hydro)	iFP	Experiment (95500)
$T_{DD}$ (keV)	21.3	15.2	$13.1 \pm 1.25$
$T_{DT}$ (keV)	19.9	17.2	$14.2 \pm 0.40$
$Y_{DD}$	$1.77 \times 10^{11}$	$1.74 \times 10^{10}$	$1.44 \times 10^{10}$
$Y_{DT}$	$3.67 \times 10^{13}$	$1.84 \times 10^{13}$	$1.54 \times 10^{13}$

The simulated neutron energy spectra can be converted into synthetic nTOF data using the detector response function<sup>1</sup> and detector backgrounds



<sup>1</sup> Z.L. Mohamed, O.M. Mannion, E.P. Hartouni, J.P. Knauer, and C.J. Forrest, *J. Appl. Phys.* 128, 214501 (2020).

# The combined algorithm provides us with over $10^{12}$ reduction in computational complexity

$$\frac{N_{v,static} N_{x,static}}{N_{v,adapt} N_{x,adapt}} = \left( \underbrace{\sqrt{\frac{v_{th,max}}{v_{th,min}}} \times \sqrt{\frac{m_{SiO2}}{m_D}}}_{\sim 300} \right)^2 \times \underbrace{\frac{\Delta x_{max}}{\Delta x_{min}}}_{\sim 100} \sim 10^7$$

$$\frac{\langle \Delta t \rangle_{HOLO}}{100 \times \Delta t_{exp}} \sim 10^5,$$

**Simulation takes <24 hours on 386 cores**

# Summary and Next Step

- Summary
  - r-adaptive (moving) phase-space grid with discrete conservation
    - Allows for more than 7 orders of magnitude saving in computational unknown in implosion problem
    - Conservation achieved via method of discrete nonlinear constraint
  - Fully nonlinear implicit time integration via HOLO nonlinear accelerator
    - Avoids the need of explicitly constructing and inverting a dense Jacobian
    - Allows one to take time-step size on order of dynamical time scale
  - The combined strategy is used to solve practical ICF implosion problem to aid experimental design
  - **Using the new capability to resolve long standing mysteries and helping design new experiments to further our understanding of HED physics**
- Ongoing work
  - Kinetic electrons with Maxwell's equations in spherically imploding system to model non-local heat flux from critical surface into capsule (transfer of laser energy into fuel): Steven E. Anderson
  - Coupling radiation transport (Boltzmann-photon) with plasma: Hans R. Hammer
  - 3D3V extension to investigate geometric effects: William T. Taitano

# **Extension to 3D3V**

# Extension to 3D3V: We adopted a hybrid coordinate system

- Transformed Vlasov and fluid electron equation (for now; extension to Fokker Planck in planning):

$$\frac{\partial(\mathbb{J}f)}{\partial t} + \nabla_{\xi} \cdot [\mathbb{J}\bar{J}_{\xi}(\vec{v}_s - \vec{w}_g)f] + \nabla_{\hat{v}_s} \cdot [\mathbb{J}\bar{J}_{\hat{v}_s}(\vec{a} - \vec{a}^{\dagger})f] = 0$$

$$\frac{3}{2} \left[ \frac{\partial \mathbb{J}_r P_e}{\partial t} + \nabla_{\xi} \cdot [\mathbb{J}_r(\vec{u}_e - \vec{w}_g)P_e] \right] + \mathbb{J}_r n_e \vec{u}_e \cdot \vec{E} = 0$$

- $f = f(\vec{\xi}, \vec{v}_s, t)$ : Distribution function
- $\bar{J}_{\xi} = \frac{\partial \vec{\xi}}{\partial \vec{r}}$ : Contravariant Jacobian matrix for configuration block and  $\mathbb{J}_r = (\det|\bar{J}_{\xi}|)^{-1}$
- $\bar{J}_{\hat{v}_s} = \frac{\partial \vec{\hat{v}}_s}{\partial \vec{v}}$ : Contravariant Jacobian matrix for velocity block and  $\mathbb{J}_v = (\det|\bar{J}_{\hat{v}_s}|)^{-1}$
- $\mathbb{J} = \mathbb{J}_{\xi} \mathbb{J}_{\hat{v}_s}$ : Composite Jacobian
- $\vec{\xi} = \xi^1 \hat{e}_1 + \xi^2 \hat{e}_2 + \xi^3 \hat{e}_3$ : Cartesian logical space [i.e., physical coordinates are represented as  $\vec{r} = \vec{r}(\vec{\xi}, t)$ ]
- $\vec{v}_s = v^* \hat{v} [S_{\theta_p} C_{\phi_p} \hat{e}_x + S_{\theta_p} S_{\phi_p} \hat{e}_y + C_{\theta_p} \hat{e}_z]$ : Proj. of part. vel (sph coord) into the Cart. lab-frame [ $v^* = v^*(\vec{\xi}, t)$  is the normalization speed]
- $\vec{w}_g = \frac{\partial \vec{r}}{\partial t}$ : Grid velocity
- $\vec{a} = \frac{q}{m} (\vec{E} + \vec{v} \times \vec{B})$ : Lorentz force in the Cart. Coord. Simplifies momentum conservation. (note,  $\vec{B}$  is an external magnetic field in this work).
- $\vec{a}^{\dagger} = \frac{1}{v^*} \frac{\partial \vec{v}_s}{\partial t} + \frac{(\vec{v}_s - \vec{w}_g)^T \cdot \frac{\partial v_s}{\partial \xi}}{v^*}$ : Inertial terms due to velocity grid expansion and contraction (mean shifting also in plans)

# Advantages of the hybrid representation

- Configuration Space: Moving mapped (non-orthogonal) grid
  - Allows for geometric conservation laws to be trivially ensured for arbitrary coordinates
- Velocity Space: Normalize spherical coordinates (laboratory frame)
  - Allows for  $\vec{v} \cdot (\vec{v} \times \vec{B}) = 0$  to automatically be ensured
  - Naturally adapts velocity domains to the heating/cooling plasmas
- A composite transformation with block diagonal Jacobian matrix:

$$-\bar{\bar{J}} = \begin{bmatrix} \bar{\bar{J}}_r & \bar{\bar{0}} \\ \bar{\bar{0}} & \bar{\bar{J}}_v \end{bmatrix} \quad \mathbb{J} = (\det|\bar{\bar{J}}|)^{-1} = \mathbb{J}_r \mathbb{J}_v$$

- Allows for an efficient decoupling of grid adaptation strategy between  $\vec{\xi}$  and  $\vec{v}$  space

# Geometric Conservation Law (GCL) in Configuration Space

- Consider an advection equation for a scalar,  $\phi$ , with a velocity field,  $\vec{v} = v\vec{C}$  where  $\vec{C} \in \mathbb{R}^3$  is an arbitrary, constant vector (e.g., particle direction):

$$\frac{\partial(\mathbb{J}_r\phi)}{\partial t} + \nabla_\xi \cdot [\mathbb{J}_r(\vec{v} - \vec{w}_g)\phi] = 0$$

- Assuming  $\partial_t\phi = 0$ , and  $\nabla_\xi\phi = 0$ , and since  $\vec{v}$  is divergence-free  $[\nabla_\xi \cdot (\mathbb{J}_r\vec{v}) = 0]$ :

$$\frac{\partial \mathbb{J}_r}{\partial t} = \nabla_\xi \cdot [\mathbb{J}_r \vec{w}_g]$$

- The above relation is typically referred to as the geometric conservation law in the literature [7].
- The inability to satisfy this property leads to false (numerical) dynamics polluting solution [8].

# Advantage of spherical geometry representation of velocity space

- Trivial representation of Maxwellian distribution (rotationally invariant)

$$\partial_{\theta_p} f_M = \partial_{\phi_p} f_M = 0$$

- Automatic enforcement of zero work constraint of Lorentz force,  $\vec{v} \cdot (\vec{v} \times \vec{B}) = 0$ :

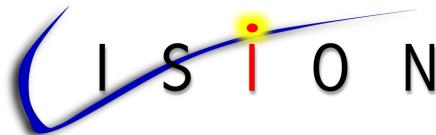
$$\begin{aligned}\nabla_{v_x} \cdot [(\vec{v} \times \vec{B})f] \mathbb{J}_{v_s} &= \nabla_{v_s} \cdot [\mathbb{J}_{v_s} \bar{J}_{v_s} (\vec{v}_s \times \vec{B})] \\ &= \partial_{\theta_p} [\mathbb{J}_{v_s} \dot{\theta}_p f] + \partial_{\phi_p} [\mathbb{J}_{v_s} \dot{\phi}_p f]\end{aligned}$$

i.e., a simple rotation operator in the velocity space. Thus, we discretely enforce the zero work constraint automatically:

$$\left\langle v, \partial_{\theta_p} [\mathbb{J}_{v_s} \dot{\theta}_p f] + \partial_{\phi_p} [\mathbb{J}_{v_s} \dot{\phi}_p f] \right\rangle_{\delta v_s} = 0$$

# V<sub>lasov</sub> | I<sub>mplosion</sub> SION

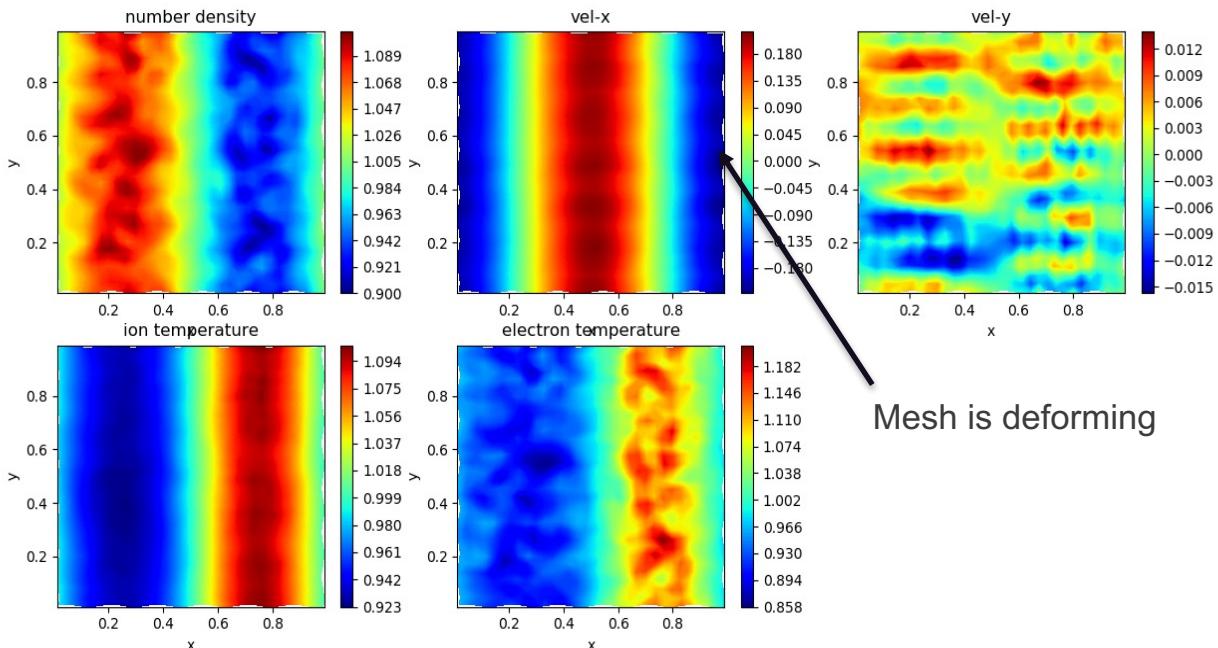
- The new capability is being implemented into the VISION code, written with the Julia language
  - Allows for rapid development and high performance.
  - All results shown were generated in only 7 weeks of dev. time.
- 3D3V, fully conservative (mass, momentum, energy), hybrid grid, implosion capability, and planned to be electromagnetic. Plenty of lessons learnt from iFP applied (expedited development time-scale)
- MPI wrapper to Julia for distributed parallelism



# Hybrid (with electrons) electrostatic test with conservation

- **IC**

- $n_0 = n(\vec{\xi}, \tau = 0) = 1 + \text{rand} * 0.5 \sin(2\pi x)$
- $\vec{u}_0 = \vec{u}(\vec{\xi}, \tau = 0) = 0$
- $T_0 = T_{e,0} = T(\vec{\xi}, \tau = 0) = 1$
- $f_0 = \frac{n_0}{(2\pi T_0)^{\frac{3}{2}}} \exp\left[-\frac{(\vec{v} - \vec{u}_0)^2}{2T_0}\right]$



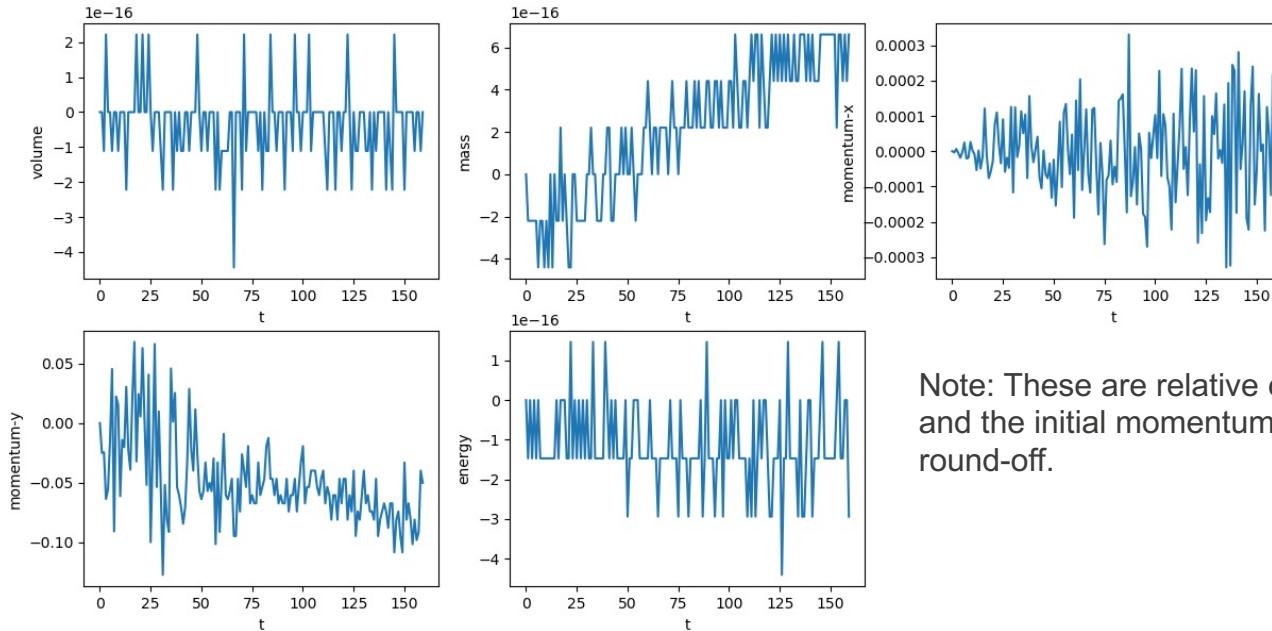
- **BC**

- Periodic in x and y

- $x = x(\vec{\xi}, \tau), y = y(\vec{\xi}, \tau)$ 
  - $x = x_0 + \frac{\Delta x_0}{5} \sin k_1 \xi^2 \sin \omega_g t$
  - $y = y_0 + \frac{\Delta y_0}{5} \sin k_2 \xi^1 \sin \omega_g t$

# Conservation is ensured to round-off per theorem

$$\mathbb{E} = \frac{\langle (\mathbb{J}_r \phi)^{(t)} - (\mathbb{J}_r \phi)^{(0)} \rangle_x}{\langle (\mathbb{J}_r \phi)^{(0)} \rangle_x}$$

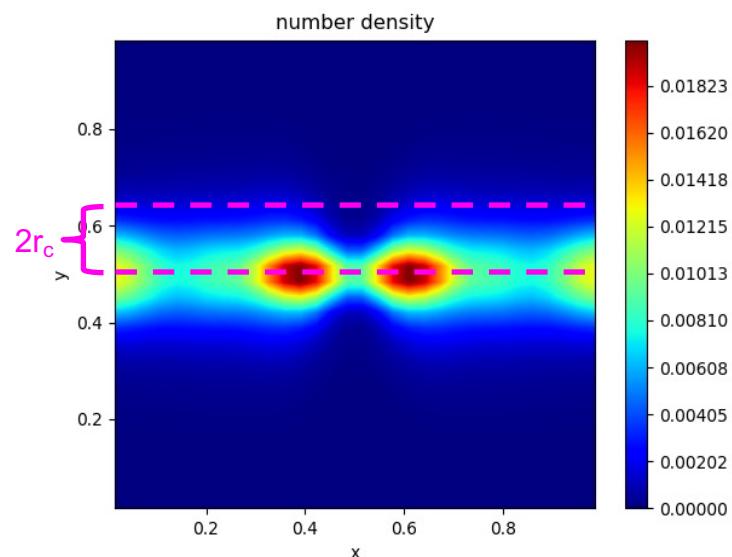
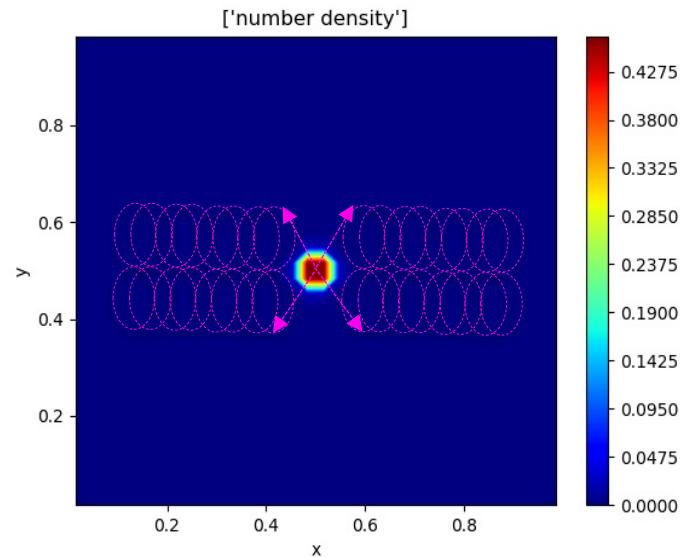


Note: These are relative errors  
and the initial momentums are  
round-off.

# Gyrating blob (no electrons, in-plane magnetic field)

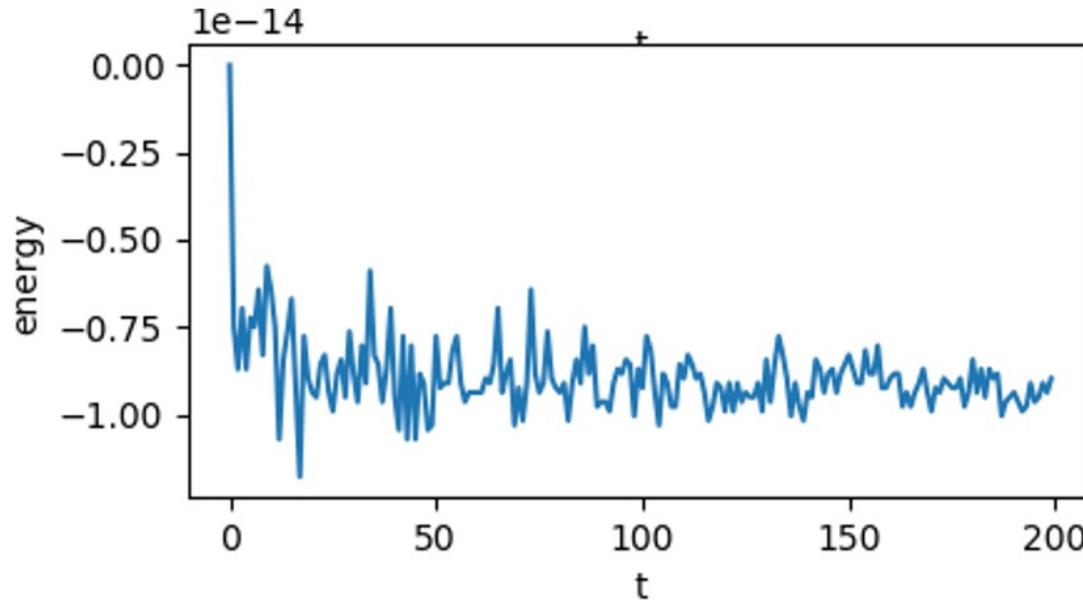
- I.C

- $f_0 = f(\vec{x}, \vec{v}, \tau = 0) = \frac{\delta(x-0.5, y-0.5)}{(2\pi T_0)^{\frac{3}{2}}} \exp\left[-\frac{v^2}{2T_0}\right]$
- $T_0 = T(\vec{x}, \tau = 0) = 0.1; B_x = 5 \leftarrow 2r_c = \frac{2mv_{th}}{qB} \sim 0.1265$

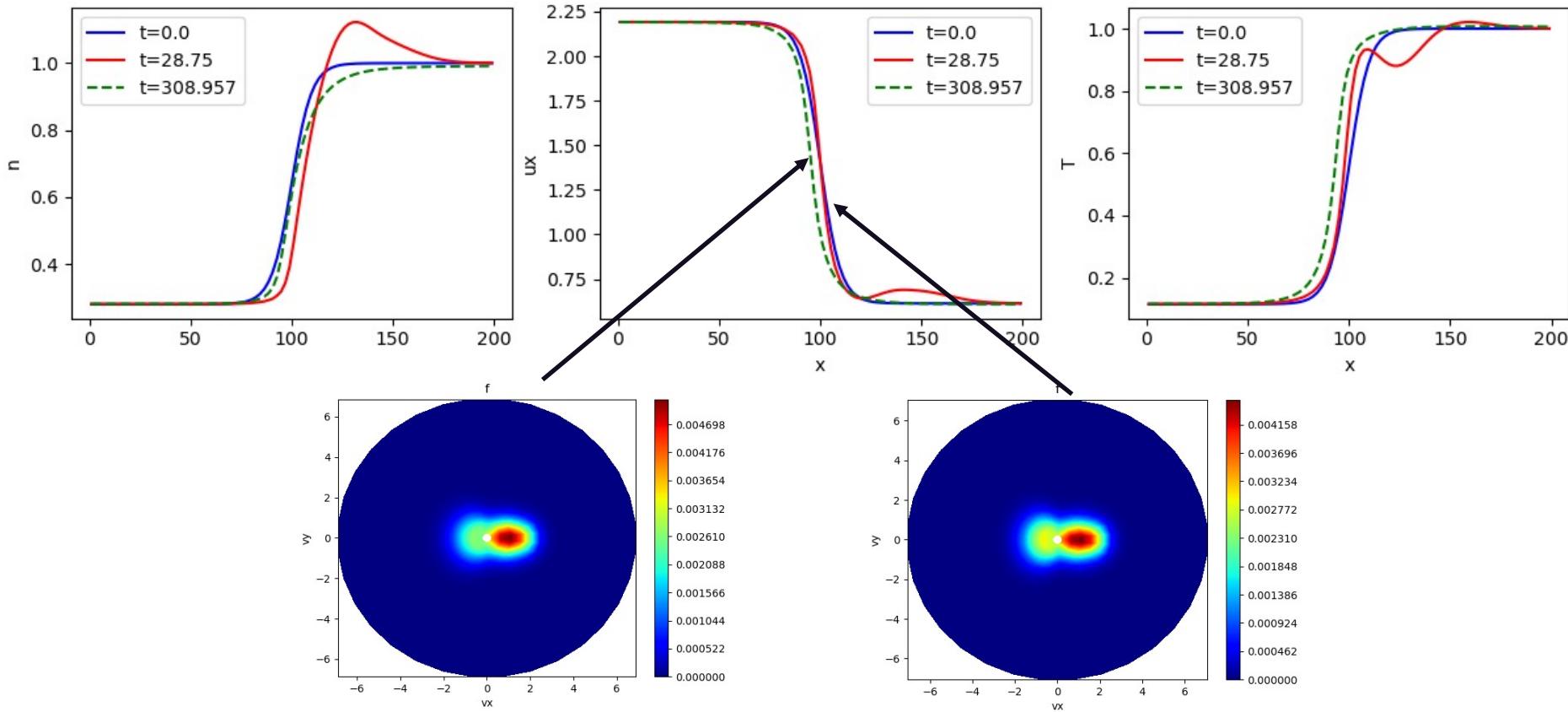


$\int d^3v \frac{v^2}{2} \nabla_v \cdot (\vec{v} \times \vec{B}) = -\int d^3v \vec{v} \cdot \vec{v} \times \vec{B} = 0$  is trivially ensured in spherical velocity space geometry

$$\mathbb{E} = \frac{\langle (\mathbb{J}_r U)^{(t)} - (\mathbb{J}_r U)^{(0)} \rangle_x}{\langle (\mathbb{J}_r U)^{(0)} \rangle_x}$$



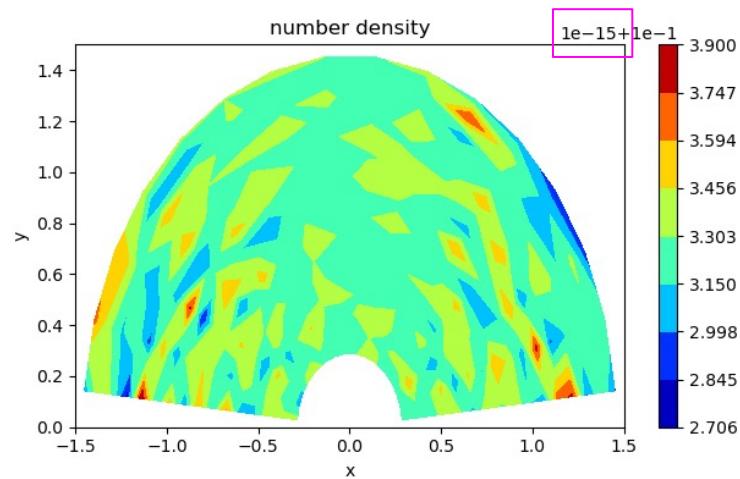
# Mach 5 shock with BGK collision operator (no electrons): Correct Hugoniot jump conditions recovered (discrete conservation enforced)



# GCL test with Cylindrical Implosion & Dirichlet BC

GCL is satisfied to round-off with everything turned on  
(streaming, moving phase-space grid, collisions, electrons, etc.)

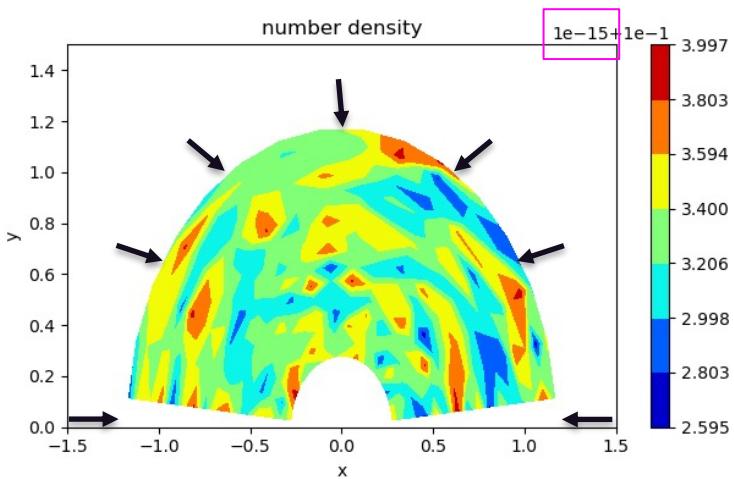
Initial Condition



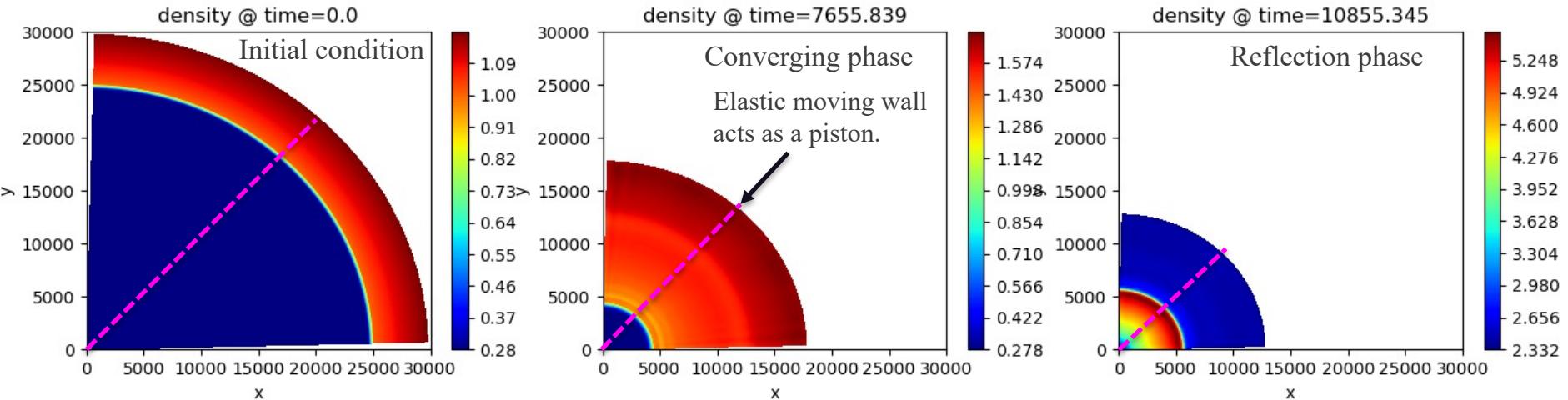
implosion



Later time

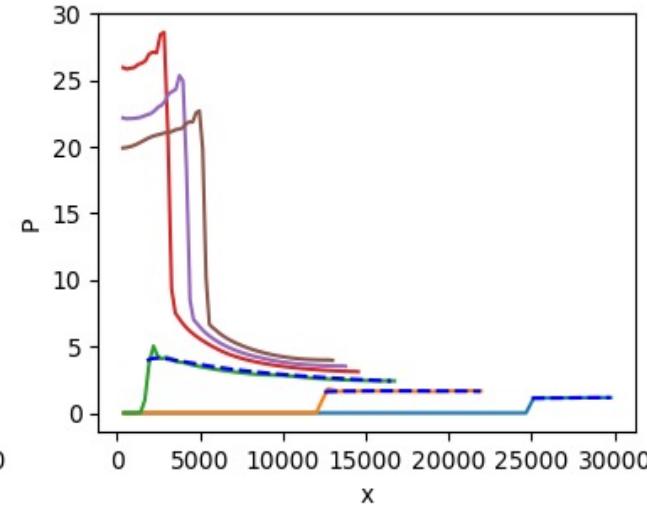
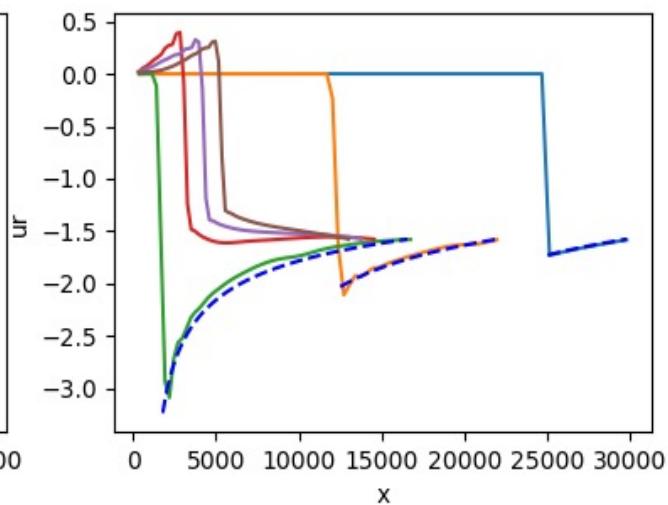
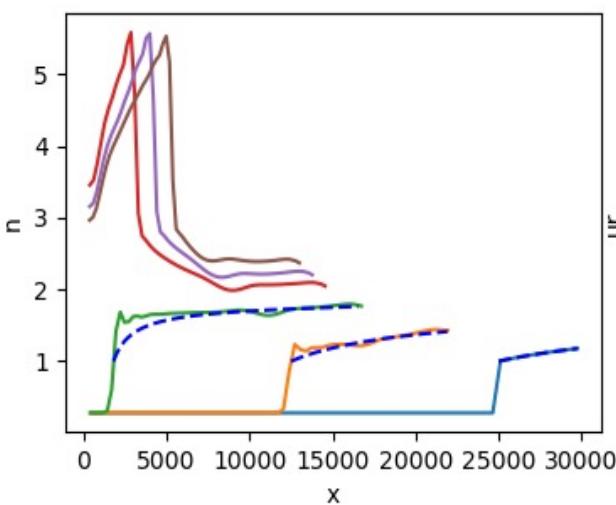


# Cylindrical Van-Dyke/Guderley Problem (no electrons): Symmetric Implosion achieved without significant grid imprinting.



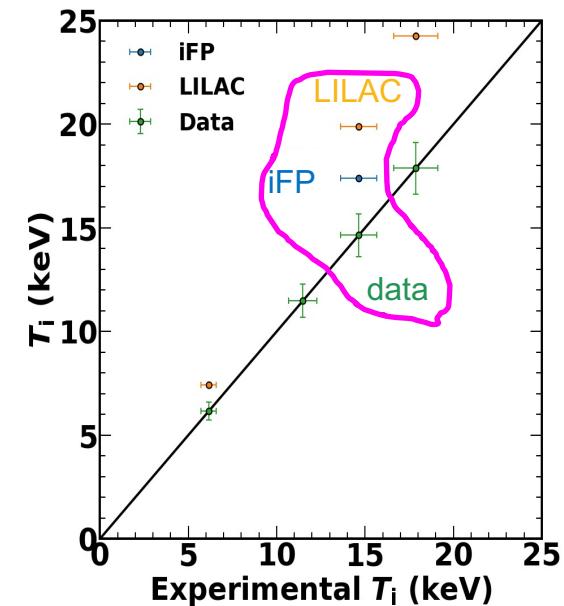
# Decent agreement with semi-analytical Guderley solution in cylindrical geometry.

Dashed blue lines: Guderley solution for converging shock

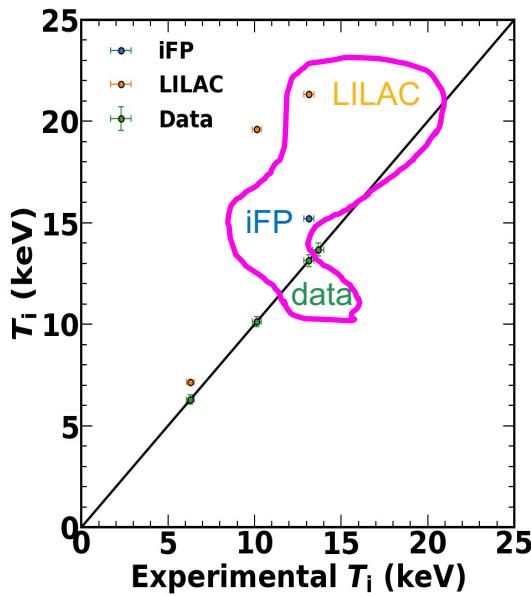


# kinetic effects captures order of magnitude correction in yield and brings inferred temperature closer to experiment

DT inferred temperature



DD inferred temperature



Obs.	LILAC (Rad hydro)	iFP	Experiment (95500)
$T_{DD}$ (keV)	21.3	15.2	13.1
$T_{DT}$ (keV)	19.9	17.2	14.2
$Y_{DD}$	$1.77 \times 10^{11}$	$1.74 \times 10^{10}$	$1.44 \times 10^{10}$
$Y_{DT}$	$3.67 \times 10^{13}$	$1.84 \times 10^{13}$	$1.54 \times 10^{13}$